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MATRIC NO: 15/ENG02/021

SUBJECT: ENG381

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Answer

$m^2 - 4 = 0$

$m^2 = 4 \Rightarrow m = \pm\sqrt{4}$

$m = \pm 2$

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

substituting

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = \frac{10e^{3x}}{5e^{3x}}$

$C = 2$

$\therefore P.I = y = 2e^{3x}$

$G.S = C.F + P.I$

$y = e^{2x}(A + Bx) + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Answer

$m^2 + 2m + 1 = 0$

$m^2 + 1m + 1m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1) = 0$ twice

$m = -1$ twice

P.I $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$P.I = 1e^{-2x}$

$G.S = C.F + P.I$

$y = e^{-1x}(A + Bx) + 1e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\Rightarrow$ $\cancel{d^2y}$ $m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm j5$

$f(x) = 5x^2 + x$

$PI = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + \cancel{E}) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 2C + 25E = 5x^2 + x$

$25Cx^2 = 5x^2$

$25C = 5$ — (i) $C = \frac{1}{5}$ — (ib)

$25Dx = x$

$25D = 1$, $D = 1/25$

$2/125$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$\cancel{25}E = \frac{2/5}{25}$, $E = 2/5 \times \frac{\cancel{25}}{25}$ $E = \cancel{00}\ 2/12$

$\therefore C = 1/5,\ D = 1/25$

Cont.d

$$P.I = \frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}$$

$G.S = C.F + P.I$

$y = A\cos nx + B\sin nx$

$$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}$$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

<u>Solution</u>

$m^2 - 2m + 1 = 0$

$m^2 - 1m - 1m + 1 = 0$

$(m^2 - 1m) \mp (1m \mp 1) = 0$

$m(m-1) \mp 1(m-1) = 0$

$(m-1)(m \mp 1) = 0$

$m_1 = 1, \quad m_2 = 1$ (twice)

$f(x) = 4\sin x$

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$\Rightarrow [-C\cos x - D\sin x] - 2[-\sin x + D\cos x] + [C\cos x + D\sin x] = 4\sin x$

$\Rightarrow [-C\cos x - D\sin x] + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

Like terms

$\Rightarrow \cos x[-C \mp 2D + C] + \sin x[-D + 2C + D]$

$= 4\sin x$

$\Rightarrow \cos x[-2D] + \sin x[2C] = 4\sin x$

comparing coefficients

$\Rightarrow -2D = 0$ —(1)

$D = -0/2 \Rightarrow D = 0$

$\Rightarrow 2C = 4$ —(2)

$C = 2$

$P.I = 2\cos x + 0\sin x$

$y = 2\cos x$

$G.S = y = e^{x}[A + Bx] + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Answer

$m^2 + 4m + 5 = 0$

using almighty formula

$\Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$\Rightarrow \frac{-4 \pm \sqrt{16 - 20}}{2} \Rightarrow \frac{-4 \pm \sqrt{-4}}{2}$

$\Rightarrow \frac{-4 \pm j\sqrt{4}}{2} \Rightarrow \frac{-4 \pm j2}{2}$

$\Rightarrow -2 \pm j$

$\Rightarrow -2 \pm j$

$$y = e^{-2x}(A\cos x + B\sin x)$$

$$F(x) = 2e^{-2x}$$

$$P \cdot I;\ y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$\Rightarrow 4Ce^{-2x} + 4[-2Ce^{-2x}] + 5[Ce^{-2x}] = 2e^{-2x}$$

$$\Rightarrow 4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$\Rightarrow Ce^{-2x} = 2e^{-2x}$$

$$\Rightarrow C = 2$$

$$P \cdot I;\ y = 2e^{-2x}$$

at $x = 0,\ y = 1$

$$1 = e^{-2(0)}(A\cos(0) + B\cos(0)) + 2e^{-2(0)}$$

$$1 = 1(A + 0) + 2$$

$$\Rightarrow 1 = A + 2$$

$$A = -2 + 1$$

$$A = -1$$

at $x = 0,\ dy/dx = -2$

$$dy/dx = -2e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}$$

$$-2 = -2e^{-2(0)}(-A\sin(0) + B\cos(0)) - 2e^{-2(0)}$$

$$-2 = -2(0 + B) - 2$$

$-2 = -2B - 2$

$-2B = -2 + 2$

$B = 0/-2$

$B = 0$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m+1) = 0$

$3m + 1 = 0$

$3m = -1$ or $m - 1 = 0$

$m_1 = -1/3$ or $m_2 = 1$

$C.F = y = Ae^{-1/3 x} + Be^{x}$

$P.I:\ y = Cx + D$

$dy/dx = C$

$\frac{d^2y}{dx^2} = 0$

Substitutions

$3(0) + -2(C) - (Cx + D) = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

$-2C - D - Cx = 2x - 3$

Comparing coefficcients

$-2C - D = -3$

$-C = 2$

$C = -2$

for D,

$-2(-2) - D = -3$

$4 - D = -3$

$\cancel{4}4 + 3 = D$

$D = 7$

$P.I = y = -2x + 7$

$G.S = C.F + P.I$

$= y = Ae^{-1/3x} + Be^{x} + (-2x + 7)$

$= Ae^{-1/3x} + Be^{x} \cancel{+} - 2x + 7$

8) $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 8e^{4x}$

Solution

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$(m^2 - 2m) + (-4m + 8) = 0$

$m(m-2) - 4(m-2) = 0$

$m - 4 = 0$ or $m - 2 = 0$

$m_1 = 4$ or $m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

P.I : $y = Ce^{4x}$

$dy/dx = 4Ce^{4x}$

$d^2y/dx^2 = 16Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$Ce^{4x}(16 - 24 + 8) = 8e^{4x}$

$Ce^{4x}(0) = 8e^{4x}$

$C = \frac{8e^{4x}}{0e^{4x}}$

$C = 0$

$y = 0e^{4x}$

G.S ; C.F + P.I

$y = Ae^{4x} + Be^{2x}$